OPTIMIZATION OF THE GROOVE CAM MECHANISM

Ha NGUYEN VAN[1], Ladislav SEVCIK[2]

[1]Department of Manufacturing Technology of Hung Yen University of Technology and Education, Viet Nam
[2]Department of Design of Elements and Mechanism of Technical University of Liberec,Czech Republic

Abstract
The article deals with the optimization of the groove shapes of the output shaft and the optimization of the shape of the groove side surfaces and the overall design of the middle cam mechanism member to reduce contact pressure. The optimization process results in a reduction of the maximum contact pressure to 863 MPa when the recommended value is up to 800 MPa. It can thus be stated that this embodiment is already practically applicable.

Key words: *Ball cam; cam groove optimization; contact analysis; novel cam; groove ball transmission.*

INTRODUCTION

Cam mechanisms are widely used in many types of modern machines because of their excellent properties for operation speed, motion accuracy, structural rigidity, and low production cost. Generally, plate cam mechanisms are only one of the larger number of the cam and follower combinations and can be classified in several ways.

Norton (2002) and Rothbart (2004) present a method for classifying a cam mechanism called the force-closed and form-closed cam mechanisms. For kind of force-closed cam mechanisms they describe that in order to ensure constant contact between the cam and follower, an external force is required to be applied. This force is usually provided by a spring of sufficient stiffness or sometimes by an air cylinder. Therefore, as a result, of the force effect, the driving torque and contact stress between the cam and follower in the rise are increased, which cause to make the wear on the cam and follower. Whereas, in the form-closed cam mechanism, contact is obtained by letting the roller follower run in a cam groove or by using a conjugate condition. Thus no force is required to maintain the contact between its cam and the follower. On the other word, the contact stress between the cam and the follower will consequently be smaller and the driving torque in the rise can be reduced.

In recent years, with the rapid development and popularization of mechanical products in modern society, which has been such an important question on designing and producing mechanisms precisely. Besides, with the assistance of the numerical controlled manufacture system being widely applied in this time, which are useful tools for the designer to develop a new type of cam profile.

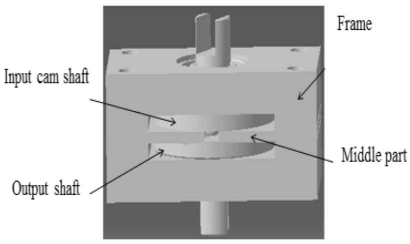
Therefore, a novel cam mechanism (Fig. 1) is proposed to design for converting a cam rotation to a desired output motion of the output shaft. The mechanism is composed mainly of a rotary input camshaft, a rotary output shaft, a middle part, and two balls and a frame to mount all the parts. In previous work *(Nguyen & Sevcik, 2017b; Nguyen, 2019)*, the author of this paper presented the first design model of the groove cam mechanism. The mechanism includes the input camshaft and the output shaft, which were described in Fig. 2 and Fig . 3.

Fig. 2 describes the shape of the input camshaft, where on one of the tops of the input camshaft has an eccentric circular groove is created with a groove radius is 3 mm, the eccentric is 6mm (the distance length between the center of the shaft axis and the center of the circular groove), the radius of the circular groove from center of rotation is 20 mm.

Fig. 3 depicts the shape of the output shaft. Like as the input camshaft, on one of the tops of the output shaft, which is designed two straight grooves. These straight grooves must have the same groove radius with the circular groove of the input camshaft. And the number of the straight grooves on the output shaft can be designed depends on the requirements of the design.

The main function of the balls is the transfer the moments and velocities from the input camshaft into the output shaft. So when cam works each ball can be easy to roll up and down with respect to their

straight groove on the middle part and the output shaft correspondingly. Moreover, at the same time, these balls must roll along in the circular groove of the input camshaft.
Due to each steel ball only rolls up and down in each straight groove of the middle part. Which plays the role of a roller follower of a cam mechanism. Therefore, the groove cam mechanism model is designed with a pair of followers for one cam but it is not a type of conjugate cam system. Due to the cam mechanism is designed which takes a form-closed, so the groove cam mechanism gets some advantages to other cam mechanisms because of following considerations: compared with a force-closed cam mechanism, the mechanism withstands lower contact stress and no force is required to maintain the contact between its cam and the follower; compared with conjugate cam mechanism in which a set of two cams must be used, the cam mechanism is simpler in construction because only one cam is needed. So, it occupies a small space and has a lower cost.



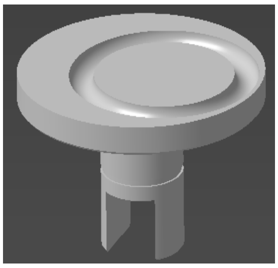
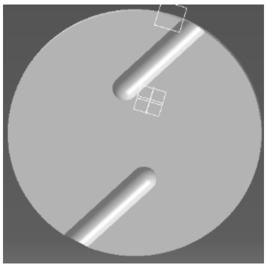



Fig. 1 Groove am mechanism model **Fig. 2** The input cam shaft **Fig. 3** The output shaft of the first design model

Another point in its favor of this groove cam model which used the ball followers, where the ball is commonly applied in the bearing and ball screw fields. Therefore, which is easy for replacement and availability from bearing manufacturers stock in any quantities are advantages. Moreover, in recent years, the idea uses the ball for gearing design has been developing by Sincroll Company *(Johnson, 1985).*
By the technology, the roller or balls connect the wheels or bodies, while the performing pure rolling motion without sliding. Thus, the power efficiency is higher, the sliding friction is approximately eliminated and so on. Therefore, the idea of altering the roller follower by the balls is applied for a groove cam mechanism in this research.
By this design, the surface contacts between the ball with their grooves are always changing when cam working. This contact phenomenon could cause to make the balls can damage easily and the result is the failure of these balls *(Gopalakrishnan &Ruban, 2015).*Therefore, the task to analyze the contact pressure between the ball and their grooves must pay attention to this research. Based on the results of contact calculation obtained by using the analytical calculation method and FEM for the first design model *(Nguyen, 2019).* We knew that the resulting deviation in contact stress is 4.52 %. But if we compared these result with the lists the maximum contact pressures and their recommended values for different types of materials according to *(Chablat et al., 2007).*
Allowable pressure for refined steel is up to 2000 MPa and the maximum pressure obtained from the analyses is 2093 MPa. As recommended pressure is 800 MPa, it was necessary to modify the design of the grooves so as to lower the pressure at the contact surfaces. Therefore, the aim of this study is to design an optimized design a new cam mechanism with respect to the minimum contact pressure to reach the recommended pressure value for steel material based on changing the groove shapes of the output shaft and the middle part of the groove cam mechanism.

MATERIALS AND METHODS

In the design of the cam mechanism, the two main factors must be considered. The first one is the selection of a proper motion curve and the second one is the requirement for the radius of curvature of the cam profile to be greater than a minimum limit to avoid the undercut phenomenon. Therefore, to optimize the groove cam mechanism, the design must meet the above requirements.
In this article, we focus on how to reduce the contact stress between the ball and the groove in design to reach the maximum recommended value contact pressure for steel material, which is a significant key for optimum design of the groove cam.

In our case, when the cam mechanism works each steel ball body in contact with three grooves, which are the groove of the input camshaft, middle part, and the output shaft. The ball is a spherical shape, the groove of the input camshaft is circular and the grooves of the middle part and the output shaft are straight. Therefore, in the previous design *(Nguyen & Sevcik, 2017b; Nguyen, 2019).*
the calculated we assumed that the contact between the ball and the straight groove is like contact between sphere-on-flat plate. It may cause high contact pressure.
For a given groove cam profile, the maximum value of the Hertz pressure is obtained for the minimum radius of curvature of the groove cam. Obviously, it is easy to know that the Hertz pressure is a maximum when the contact area is a minimum value and the magnitude of the force is a maximum. Consequently, the higher contact area and the smaller force makes the Hertz pressure is lower. This problem was reviewed by Johnson (1985) *(Nguyen & Sevcik, 2017a)*. He first reviews the development of the theory of contact stresses since the problem was originally addressed by Hertz in 1882.
The radius of the contact-patch of the contact depends on several parameters, amongst them, the equivalent radius of the contact (geometry constant *B*) following equation

$$B = \frac{1}{2}\left(\frac{1}{R_f} + \frac{1}{R_r}\right), \qquad (1)$$

R_r is the radius of the ball, which is constant and R_f is the radius of the groove in contact with the ball (in this case which is either of a radius of circular groove cam or the groove of the output shaft or the groove of the middle part). Therefore, B depends only on R_f. Finally, for a given cam profile, a_h is a minimum when R_f is a minimum. Hence, to compute the maximum value of the Hertz pressure, we have to consider the lowest value of R_f.
Obviously, the maximum value of the contact pressure depends on several parameters, namely, the shapes of the parts in contact, the number of the groove of the output shaft, the material of the parts in contact, the load applied. Therefore, we have some different ways to minimize Hertz pressure:

i) Increase the number of the groove of the output shaft;
ii) Decrease the minimum value of the radius of the R_f by changing the shape of the groove cam as well as the groove shape of the output shaft and middle part;
iii) Decrease the load applied;
iv) Choose a material with a lower Young modulus;
v) Together with change the shape of the frame to make the mechanism is more stable.

Based on the result of the previous studied presented by own author *(Nguyen, 2019)* combines with stated suggestions above, in this section, we can choose the way to optimize the groove cam mechanism following:
Firstly, we changed the straight groove by a curved groove, also changing the shape of the frame.
Secondly, Remained the curved groove of the output shaft in the previous case and changed the flat surface of the straight groove of the middle part by spherical surface. By this replacement, the shape of the ball and the groove of the middle part is the same size. Therefore, it may increase more elements in contact between the ball and the groove of the middle part. Hence, the maximum contact pressure may also reduce.

RESULTS AND DISCUSSION

1. Optimization of the groove cam by changing the groove shape of the output shaft. (the second design model)

To carry out this design we must follow some procedures below.
Find the trajectory of the groove of the output shaft we use inverse model and coordinate systems to analysis follow
The reverse model is used to translate the relative motion of follower with respect to cam into absolute motions, that is, let the cam stationary and the follower and the frame rotate at the same velocity as the cam but in the direction opposite to the cam at the same time, the follower reciprocates in the predefined motion with respect to the frame.
Fig. 4 is a generalized model of the groove cam mechanism, the cam rotates counterclockwise at a constant angular velocity and drives the output shaft. To express the motion of each part in a groove cam, three coordinate systems are set. The first is called a Cam coordinate system (CCS) $X_1O_1Y_1$ with an origin fixed at the axis of the cam to describe the cam shape. The second is a frame coordinate sys-

tem (FCS) XOY with an origin located on the cam axis and its Y-axis normal to the path of translating follower (the trajectory of the ball center in the straight groove of the middle part). This coordinate system coincides with the coordinate system of the middle part. This coordinate describes the position and motion of the ball with respect to the cam. In an initial position where the rotational angular θ is 0 degree; the coordinate axis OX coincides with the axis O_1X_1. The third is an output coordinate system (OCS) $X_2O_2Y_2$ with an origin fixed at the axis of the output shaft used to describe the shape of the groove of the output shaft. In reality, the CCS and OCS are movable, and the FCS is stationary.

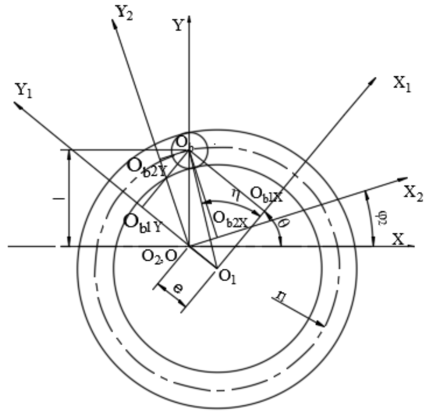


Fig. 4 Generalized model of a groove cam

Vectors are used to represent kinematical quantities and expression such as position. From Fig .4, the position of the ball's center O_b can be expressed in vector form as

$$\overrightarrow{OO_b} = \overrightarrow{OO_1} + \overrightarrow{O_1O_b}. \quad (2)$$

The vector OO_b in the frame coordinate system (FCS) as

$$OO_b{}^o = \begin{bmatrix} 0 \\ l \end{bmatrix}, \quad (3)$$

also, we have the vector OO_1 and O_1O_b in the frame coordinate system (CCS) as

$$OO_1{}^1 = \begin{bmatrix} 0 \\ -e \end{bmatrix}, \quad (4)$$

$$O_1O_b{}^1 = \begin{bmatrix} r_1 \cdot cos\eta \\ r_1 \cdot sin\eta \end{bmatrix}, \quad (5)$$

With l is the distance between the center O and the ball's center at the initial position and e is the eccentric distance between the rotating center of the cam and the center of the circular groove.

Substituting equations 4 and 5 into equation 2 obtained

$$OO_b{}^1 = \begin{bmatrix} r_1 \cdot cos\eta \\ r_1 \cdot sin\eta - e \end{bmatrix}. \quad (6)$$

Hence, the vector OO_b in the frame coordinate system (FCS) obtained

$$OO_b{}^o = R_{z,\theta} \cdot OO_b{}^1 = R_{z,\varphi_2} \cdot OO_b{}^2, \quad (7)$$

with $R_{z,\theta}$, R_{z,φ_2} are rotating matrices of the z-axis with respect to θ and φ_2 respectively, and can be expressed following

$$R_{z,\theta} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}, \quad (8)$$

and $R_{z,\varphi_2} = \begin{bmatrix} cos\varphi_2 & -sin\varphi_2 \\ sin\varphi_2 & cos\varphi_2 \end{bmatrix}$. (9)

Substituting equations 3 and 7 into 6 we got

$$\begin{bmatrix} 0 \\ l \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} r_1 \cdot cos\eta \\ r_1 \cdot sin\eta - e \end{bmatrix} = \begin{bmatrix} r_1 \cdot \cos(\theta + \eta) + e \cdot sin\theta \\ r_1 \cdot sin(\theta + \eta) - e \cdot cos\theta \end{bmatrix}. \quad (10)$$

Equation 9 can rewrite by as

$$r_1{}^2 = e^2 \cdot sin^2\theta + (l + e \cdot cos\theta)^2 = l^2 + 2 \cdot l \cdot e \cdot cos\theta + e^2 - r_1{}^2 = 0.$$

In this case the trajectory of the groove of output shaft undefined, therefore, at the initial position we assumed that the output coordinate system (OCS) situated like as Fig. 4.

If we want to design the trajectory of the groove is curved, for a simplification case we assumed that the curve may a circle and can be expressed by an equation in (OCS) as

$$(x-a)^2+(y-b)^2={R_2}^2, \tag{11}$$

With a, b are coordinates of the circle center and R_2 is the radius of the circular groove of the output shaft.

We can express the position of A in the (OCS) following

$$OA^2=\begin{bmatrix} l\cdot sin\varphi_2 \\ l\cdot cos\varphi_2 \end{bmatrix}, \tag{12}$$

with φ_2 is the rotational angle of the output coordinate system.

Substituting the values of x and y from equation 11 into 10 we obtained

$$(l\cdot sin\varphi_2-a)^2+(l\cdot cos\varphi_2-b)^2={R_2}^2. \tag{13}$$

Rewrite equation 12 got

$$2\cdot l\cdot(asin\varphi_2+bcos\varphi_2)={R_2}^2-l^2-a^2-b^2, \tag{14}$$

Denoted $sin\mu=\frac{a}{\sqrt{a^2+b^2}}$ and $cos\mu=\frac{b}{\sqrt{a^2+b^2}}$. (15)

Substituting equation 15 into 14 obtained

$$2\cdot l\cdot(sin\mu\cdot sin\varphi_2+cos\mu\cdot cos\varphi_2)=\frac{{R_2}^2-l^2-a^2-b^2}{\sqrt{a^2+b^2}}, \tag{16}$$

Finally, the obtained equation following as

$$2\cdot l\cdot(cos\varphi_2-\mu)=\frac{{R_2}^2-l^2}{\sqrt{a^2+b^2}}-\sqrt{a^2+b^2}. \tag{17}$$

For as simplification case of design, given a value of parameters then solve the equation 16 to find R_2 or a.

An example is given l=14mm, φ_2=0 degree assumed the center of the circular groove of the output leans on the x-axis, hence, b=0. From equation 16 we can choose one of two values and the last parameter can be taken.

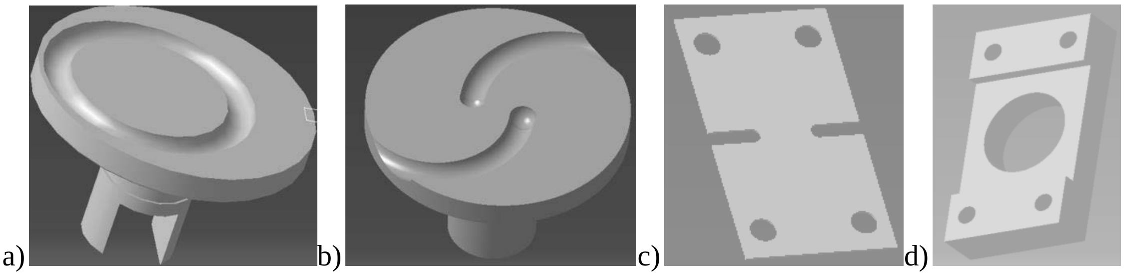


Fig. 5 a) The shape of the input camshaft, b) the shape of the output shaft, c) the middle part, d) one of two frames of the groove cam mechanism for the second design model

1.1 Calculation of contact pressure of the steel ball and the curved groove by Hertz theory

The contact between the ball and the curved groove of the output shaft can be modeled as a contact of sphere-on-cup. With the sphere is the ball and the cup is the curved groove. By applying the Hertz theory and substitution the magnitude of the design parameters we obtained the results of maximum contact pressure between the ball and their grooves.

The average and maximum contact pressure can be found respectively

$$P_{aveg}=\frac{F_{A1}}{A}=\frac{30.929}{3.678\cdot 10^{-8}}=841\text{(MPa)},$$

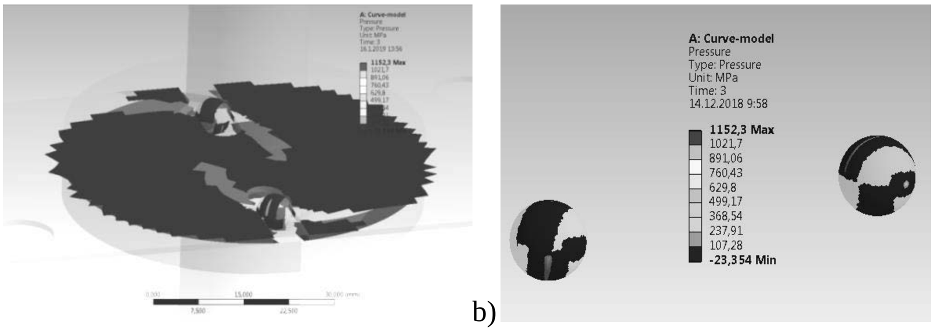
$$P_{max}=\frac{3}{2}\cdot\frac{F_{A1}}{A}=\frac{3}{2}\cdot 841=1262\text{(MPa)}.$$

1.2 Finite element analysis

The calculation was performed under the same boundary conditions as the previous cases *(Nguyen, 2019).* The material of all parts made by steel, the elasticity model is 2 E5 MPa, the Poisson's ratio is 0.3. The meshing type is free, element size is set 0.5 mm for each contact pairs. Restrained all degrees of freedom (DOF) of the middle part (fixed support), added frictionless support constraint to the input camshaft and the output shaft, and applied moment load to the input camshaft is given 1000 N.mm.

The result of calculation reveals that the maximum contact pressure reached the value of 1152 MPa, as shown in Fig. 6

From Fig. 6 we can know that the contact pressure change. The biggest pressure was at the contact point between the ball and the curved groove of the output shaft, and the lowest contact pressure existed at the contact between the ball and the groove of input camshaft, which was consistent with the fact *(Norton, 2002)*. From the simulation result of the maximal contact pressure was1152 MPa while the Hertz theory value was 1262 MPa. The comparison clearly showed that there was good consistency between the Hertz theory solution and the finite element solution. For more information can be seen in Tab. 1.But if compared the maximum contact pressure with the recommended value of the maximum pressure P_{max} for steel material *(Chablat et al., 2007)*, which is still larger. Therefore, we should improve the design to meet the goal of the design.
Also, Fig. 6 b the contact area between the ball and the straight groove of the middle part is the unconventional case. Contact area looks like a circular shape, which is a question for the designer because according to the Hertz theory, the contact area common is ellipse or rectangle shape. Therefore, this problem can be solved in the next section.



a) b)

Fig. 6 a) Distribution of contact pressure on the groove cam mechanism, 6 b) cloud chart about contact pressure on the ball in contact with their grooves of the groove cam

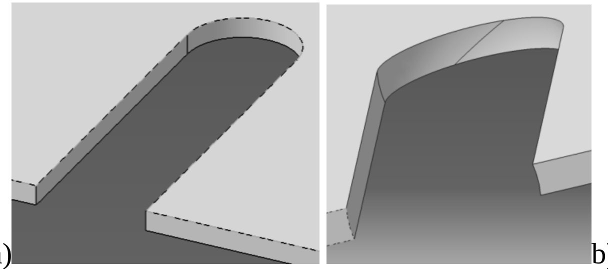
Tab. 1 Comparison pressure [MPa] between two methods

Parameter	Hertz theory solution	Finite element solution	Differences
Maximum contact pressure	1262	1152	8.7%

2. Optimization of the groove cam by changing the groove surface of the middle part (the third design model)

2.1 Redesign the surface of the straight groove

The goal of the optimal design of the groove cam mechanism is to reduce the maximum contact pressure between the ball and their grooves to approach the recommended value P_{max} (for steel material is 800 MPa). Therefore, in this section, we may think about changing the contact surface of the straight groove of the middle part, because the middle part is a thin part, and the previous cases, the contact surface of the straight groove participated in contact with the ball is a flat plate. Hence, the contact areas between the ball and the straight groove of the middle part may not enough, which may cause increasing the maximum contact pressure.
If replacing the flat plate of the straight groove of the middle part by a spherical surface, hereby the contact surface between the ball and the groove surface is the same size. Therefore, it can increase elements in contact between the ball and the groove of the middle part. Hence, the maximum contact pressure may also reduce.



a) b)

Fig. 7 a) view of the straight groove with the flat-plate surface, b) view of the straight groove with a spherical surface for the third design model

2.2 Finite element solution

For the redesigned model, all parts of the groove cam in the previous design model maintained. We had a changing the surface of the straight groove of the middle part as designed above.

The calculation was performed under the same boundary conditions as the previous cases.

By means of simulation, the contact change status can be got as Fig. 8

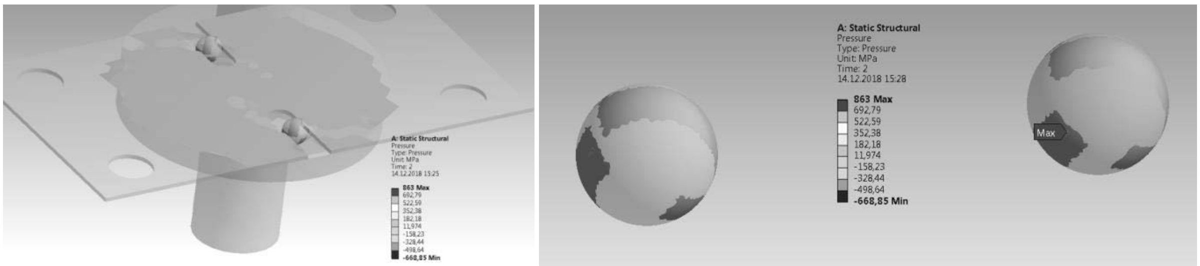


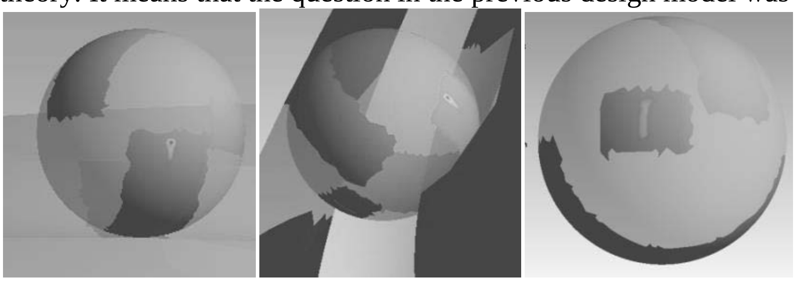
Fig. 8 Distribution of contact pressure on the groove cam mechanism for the third design model

Fig. 8 clearly shows that the maximum contact pressure, which takes the value 863 MPa, occurs at the contact point where the steel ball is in contact with the curved groove of the output shaft. For more details were presented in *(Norton, 2002).* Moreover, the maximum contact pressure is approaching the recommended value P_{max} (800 MPa). It is a very impressive number. Hence, in this case, it can be proved that the way to optimize the groove cam mechanism was successful in design.

Tab. 2 Comparison contact pressure [MPa] for redesign model and recommended value P_{max}

The redesign model	Recommended value (P_{max})	Differences
863	640 to 800	7.9%

In addition, from Fig. 9, we can clearly know that all the contact areas had an approximate ellipse shape in the contact area of the ball and their grooves, which was consistency with the Hertz contact theory. It means that the question in the previous design model was answered.



a) b) c)

Fig. 9 Cloud charts about contact pressure on the ball in contact with their grooves of the third design model a) the contact area with groove curve of the output shaft, b) the contact area with the groove of input camshaft, c) the contact area with the straight groove of the middle part

DISCUSSION

Tab. 3 Contact pressure [MPa] of all design models

Solution	Fist design model	Second design model	Third design model
Finite element	2093	1152	863

Based on the results obtained, which is obviously revealed that the maximum contact pressure of the second design model is less than the first one is 45% (refer to table 3) and the third design model is less than the second one is 25%. Therefore, the purpose of the optimal design with respect to reducing the Hertz pressure is obtained with the groove shapes of the output shaft and the middle part.

Both Hertz theory and finite element methods were applied to determine the contact pressure between the balls and their grooves of the groove cam. The results have shown that the computational values were consistent with theoretical values.

The study brought out the newest idea design for designing cam mechanism by using the ball, where the ball plays the role of a follower in the cam mechanism. Until now there are very few references can be found in the literature that addresses the issue of the application of the ball for designing the cam field. So the research may help to open the new trend for designing cam mechanism in the years coming up.

At present, the material used for calculation of this research is structural steel, which is a common material used in cam design. Nowadays with developing of the advanced material, it is necessary to expand to other material with a lower Young modulus, i.e., a more compliant material, thus increasing the surface of contact, hence, decreasing the maximum contact pressure. Moreover, when the material is more compliant, its plastic domain occurs for smaller stresses. Therefore, other material must be carried out in the future to obtain the goal of optimal design for the groove cam mechanism.

CONCLUSIONS

From the result of the numerical simulation for the first design model, which was presented in *(Nguyen, 2019)* and the last two design models which were investigated in this article. We can conclude that the maximum contact pressure between the ball and their grooves by applying the ball for cam design is approaching the recommended value P_{max} (800 MPa). Although the calculating value is slightly bigger than the allowable contact pressure of steel material, the differences are not too much (7.9%). It is a very impressive number. Thus the way to optimize the groove cam mechanism was successful in design.

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Corresponding author: Ph.D. Ha Nguyen Van., Faculty of Mechanical Engineering, Hung Yen University of Technology and Education, 39A Road, Khoai Chau District, Hung Yen province, Viet Nam, phone: +840849790998, e-mail: nguyenha.hut@gmail.com